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Effect of viscoplastic relations on the instability strain, shear band initiation strain, the strain corresponding to the minimum shear band spacing, and the band width in a thermoviscoplastic material

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Dedicated to Thomas Wright on his 65th birthday

Abstract

We study thermomechanical deformations of a steel block deformed in simple shear and model the thermoviscoplastic response of the material by four different relations. We use the perturbation method to analyze the stability of a homogeneous solution of the governing equations. The smallest value of the average strain for which the perturbed homogeneous solution becomes unstable is called the critical strain or the instability strain. For each one of the four viscoplastic relations, we investigate the dependence upon the nominal strain-rate of the critical strain, the shear band initiation strain, the shear band spacing and the band width. It is found that the qualitative responses predicted by the Wright–Batra, Johnson–Cook and the power law relations are similar but these differ from that predicted by the Bodner–Partom relation. The computed band width is found to depend upon the specimen height.

Keywords: Thermoviscoplasticity; Simple shearing deformations; Numerical solution; Perturbation method; Energy dissipation rate

1. Introduction

An interesting problem in dynamic plasticity is the determination of a constitutive relation that realistically models the response of the material for finite deformations

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over a large range of strain-rates and temperatures that are likely to occur within a shear band. A shear band is a narrow region of intense plastic deformation that usually forms when a body is deformed at high strain-rates. Batra and Kim (1990a,b,c) used Marchand and Duffy's (1988) test data on the torsion of a HY-100 steel thin-walled tube deformed at a nominal strain-rate of 3300/s to calibrate five viscoplastic relations, namely the Litonski (1977), Wright and Batra (1985), Bodner and Partom (1975), Johnson and Cook (1983) and the power law (e.g. see Klopp et al., 1985). They determined the values of material parameters by solving an initial-boundary-value problem simulating simple-shearing deformations of an homogeneous and isotropic thermo-viscoplastic body. These five constitutive relations were used to analyze the initiation and development of a shear band in a block of the same material deformed in simple shear at a nominal strain-rate of 1600/s. They found that the five constitutive relations gave essentially similar time-histories of the evolution of the shear stress, shear strain, temperature and strain-rate till the initiation of a shear band. However, the postlocalization response predicted by these constitutive relations was quite different; these and results of other numerical investigations are summarized in Batra (1998). For steady-state deformations, Wright (1987) showed that these constitutive relations predict virtually the same response. Here, we investigate, for four constitutive relations, the dependence upon the nominal strain-rate of the critical strain, the average strain at which a shear band initiates, the nominal strain corresponding to the shear band spacing and the width of a shear band formed in a HY-100 steel block which is deformed in simple shear. The thickness of the block is assumed to vary sinusoidally with the minimum thickness occurring at its center. It is found that the critical strain, defined in the abstract, monotonically increases with an increase in the nominal strain-rate for the Bodner–Partom relation but gradually decreases for the other three relations.

2. Formulation of the problem

We study simple shearing thermomechanical deformations of an isotropic, homogeneous, strain hardening, strain-rate hardening and thermally softening thermoviscoplastic body occupying the region $0 \leqslant y \leqslant h$ and being sheared in the x-direction. Equations describing the deformations are (e.g. see Batra and Kim, 1992)

$$\rho w \dot{v} = (w\tau)_{,y}, \quad \rho w c \dot{\theta} = \left(k w \theta_{,y}\right)_{,y} + \beta w \tau \dot{\gamma}, \tag{1}$$

$$\dot{\gamma} = v_{,y} = g(\tau, \gamma, \theta, \phi), \quad \frac{\mathrm{d}\phi}{\mathrm{d}\gamma} = f(\phi, \tau, \theta, \dot{\gamma}). \tag{2}$$

Here ρ is the mass density, w the thickness of the block, v the velocity of a material particle in the direction of shearing, a superimposed dot indicates the material time derivative, τ the shear stress, and a comma followed by y signifies partial differentiation with respect to y. Further, c is the specific heat, k the thermal conductivity, β the Taylor–Quinney factor describing the fraction of plastic working converted into heating, $\dot{\gamma}$ the plastic strain-rate, and θ the present temperature of a material particle. Eqs. $(1)_1$ and $(1)_2$ express, respectively, the balance of linear momentum and the balance of internal energy. We have neglected elastic deformations; this is reasonable because the plastic strains envisaged are large. Eq. $(2)_1$ gives the plastic strain-rate as a function of the shear stress, plastic strain, temperature and an internal variable ϕ whose evolution is given by Eq. $(2)_2$. The form of the functions g and f will vary with the constitutive relation employed.

For the initial conditions we take

$$v(y,0)=0,\quad \theta(y,0)=\theta_0,\quad \tau(y,0)=0,\quad \phi(y,0)=0. \tag{3}$$

That is, initially the body is at rest, is stress free, and is at a uniform temperature θ_0. The boundary conditions considered are

$$\theta_{,y}(0,t)=0,\quad \theta_{,y}(h,t)=0,\quad v(0,t)=0,\; v(h,t)=\begin{array}{ll} v_0 t/t_{\mathrm{r}}, & 0\leqslant t\leqslant t_{\mathrm{r}}, \\ v_0, & t\geqslant t_{\mathrm{r}}, \end{array} \tag{4}$$

where v_0 is the steady value of the speed assigned to the top surface of the body, and t_{r} is the rise time of the speed. For perturbation analysis, the block is assumed to be of uniform thickness and of infinite height, h, and the nominal strain-rate equals $\dot{\gamma}_{\mathrm{avg}}$.

3. Viscoplastic relations

3.1. Wright–Batra's relation

Wright and Batra (1985) generalized the relation proposed by Litonski (1977) to nonpolar and dipolar viscoplastic materials. For nonpolar materials, it is

$$\tau=\tau_0\left(1+\frac{\phi}{\phi_0}\right)^n(1+b\dot{\gamma})^m(1-\alpha(\theta-\theta_0)),\; \frac{\mathrm{d}\phi}{\mathrm{d}\gamma}=\tau(\phi,\dot{\gamma},\theta)\left(1+\frac{\phi}{\phi_0}\right)^{-n}. \tag{5}$$

The internal variable ϕ describes the work-hardening of the material; its rate of evolution is given by Eq. $(5)_2$. In Eq. $(5)_1$, τ_0 equals the yield stress of the material in a quasistatic simple shear test, parameters ϕ_0 and n characterize the work-hardening of the material, b and m its strain-rate hardening, and α its thermal softening. The parameter b may be regarded as a viscous constant for the material. A comparison of Eqs. $(5)_1$ and $(2)_1$ gives

$$g=\frac{1}{b}\left[\left(\frac{\tau}{\tau_0\left(1+\frac{\phi}{\phi_0}\right)^n(1-\alpha(\theta-\theta_0))}\right)^{\frac{1}{m}}-1\right] \tag{6}$$

provided that $\tau\geqslant\tau_0\left(1+\frac{\phi}{\phi_0}\right)^n(1-\alpha\,(\theta-\theta_0))$; otherwise $g=0$.

3.2. Power law

Klopp et al. (1985) and Marchand and Duffy (1988) have described the stress–strain curves for dynamic loading by

$$\tau = \tau_0 \left(\frac{\gamma}{\gamma_y}\right)^n \left(\frac{\dot{\gamma}}{\dot{\gamma}_0}\right)^m \left(\frac{\theta}{\theta_0}\right)^\nu \tag{7}$$

where γ_y is the strain at yield in a quasistatic simple shear test at $\dot{\gamma} = 10^{-4}$/s, parameters n and m characterize the strain and strain-rate hardening of the material and $\nu < 0$ its thermal softening. In eq. (7), θ and θ_0 are measured in Kelvin.

3.3. Bodner–Partom's relation

Bodner and Partom (1975) proposed that

$$\dot{\gamma} = D_0 \exp\left[-\frac{1}{2}\left(\frac{K^2}{3\tau^2}\right)^n\right], \quad n = \frac{a}{\theta} + b, \quad K = K_1 - (K_1 - K_0)\exp\left(-mW_p\right), \tag{8}$$

where θ is the absolute temperature of a material particle, W_p equals the plastic work done, and D_0 is the limiting value of the plastic strain-rate usually set equal to 10^8/s. A comparison of Eqs. (8) with (2) suggests that $\phi = W_p$ and $f = \tau$.

3.4. Johnson–Cook's relation

Johnson and Cook (1983) tested several materials in simple shear and simple compression at different strain-rates and temperatures, and found that

$$\dot{\gamma} = \dot{\gamma}_0 \exp\left(\left(\frac{\tau}{\tau_0(A + B\gamma^n)(1 - T^m)} - 1\right)/C\right), \; T = (\theta - \theta_0)/(\theta_m - \theta_0), \tag{9}$$

fit well the test data. Here θ_m is related to the melting temperature of the material, $\dot{\gamma}_0$ is the reference strain-rate, and parameters A, B, n, m, C, and θ_m characterize the material. For this case, $\phi = f = 0$.

4. Determination of values of material parameters

Batra and Kim (1990a,b,c) determined values of material parameters appearing in relations (5), (7), (8) and (9) by solving an initial-boundary-value problem that closely simulated the test conditions of Marchand and Duffy (1988) and ensured that the computed stress-strain curve matched well with the test data. The values of material parameters so determined are non-unique since the underlying initial-boundary-value problem need not have a unique solution. Values of the quasistatic yield stress and the strain hardening exponent were kept close to the values reported by Marchand and Duffy (1988). For the HY-100 steel,

$$\begin{aligned}&\tau_0 = 405\,\text{MPa},\ \ \dot{\gamma}_{\text{avg}} = 3300/\text{s},\ \ \rho = 7860\,\text{kg/m}^3,\ \ c = 473\,\text{J/kg}^\circ\text{C},\\&k = 49.73\,\text{W/m}^{2\circ}\text{C}\end{aligned} \tag{10}$$

and values of material parameters appearing in relations (5), (7), (8) and (9) are listed below.

(a) Wright–Batra relation:

$$\alpha = 0.00185/^\circ\text{C},\ \ \phi_0 = 4.86\ \ \text{MPa},\ \ n = 0.107,\ \ m = 0.0117,\ \ b = 10^4/\text{s}.$$

(b) Power-law:

$$\dot{\gamma}_0 = 10^{-4}/\text{s},\ \ \gamma_{\text{y}} = 0.012,\ \ \theta_0 = 300K,\ \ m = 0.0117,\ \ n = 0.107,\ \ \nu = -0.75.$$

(c) Bodner–Partom relation:

$$a = 1200K,\ \ b = 0,\ \ K_1 = 3.95\tau_0,\ \ K_0 = 3.21\tau_0,\ \ m = 5/\tau_0,\ \ D_0 = 10^8/\text{s}.$$

(d) Johnson–Cook relation:

$$\begin{aligned}&A = 0.45,\ \ B = 1.433,\ \ C = 0.0227,\ \ n = 0.107,\ \ \theta_{\text{m}} - \theta_0 = 1200^\circ\text{C},\\&m = 0.7,\ \ \dot{\gamma}_0 = 3300/\text{s}.\end{aligned}$$

For an average strain-rate of 3300/s, Fig. 1 shows the computed shear stress vs. shear strain curves with the aforegiven values of material parameters. Because of the magnified vertical scale, the differences among these curves are exaggerated. The values of the maximum shear stress and the average strain at which they occur equal (1.60, 0.25), (1.55, 0.27), (1.53, 0.27) and (1.50, 0.26) respectively for the Wright–Batra, power law, Bodner–Partom and the Johnson–Cook relations.

For high temperatures likely to occur within a shear band, material parameters in each one of the constitutive relations will depend upon the temperature. However, for the sake of simplicity, this temperature dependence has not been considered here. Also, such test data for the HY-100 steel is not readily available in the open literature.

5. Perturbation analysis

We closely follow Bai (1982) in studying the stability of the homogeneous solution of the governing Eqs. (1) and (2). Linear perturbation analysis has also been used to study the initiation of material instability by Clifton (1978) and Burns (1985) amongst others (e.g. see Bai and Dodd, 1992). Let the homogeneous solution at time $t = t_0$ be given an infinitesimal perturbation

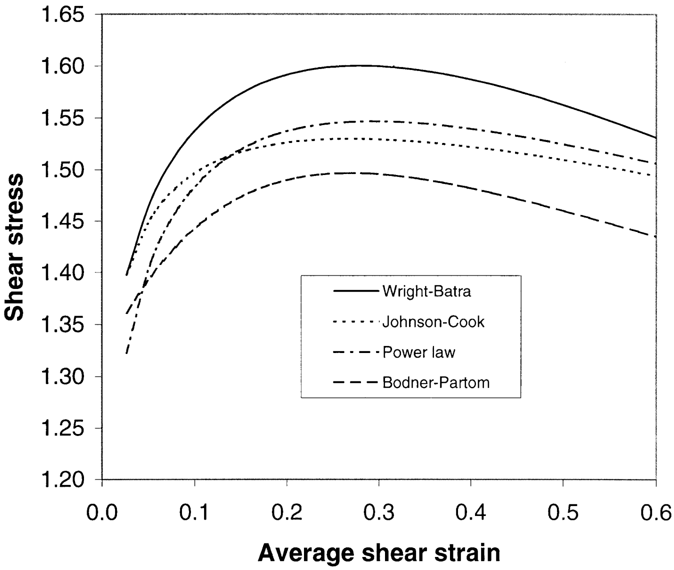


Fig. 1. Shear stress vs. shear strain curves for homogeneous simple shearing deformations of a HY-100 steel at a strain rate of 3300/s.

$$\delta \mathbf{s}(y, t, t_0) = \delta \mathbf{s}^0 e^{\eta(t-t_0)} e^{i\xi y}, \quad t \geqslant t_0, \tag{11}$$

where

$$\delta \mathbf{s}^0 = \left[\delta\gamma^0, \delta\theta^0, \delta\phi^0, \delta\tau^0\right]^T \tag{12}$$

is a small disturbance, at time t_0, in the homogeneous solution $\bar{\mathbf{s}} = \left[\bar{\gamma}, \bar{\theta}, \bar{\phi}, \bar{\tau}\right]^T$. Here ξ is the wave number and η its initial growth rate. Re $(\eta) > 0$ means that the homogeneous solution at $t = t_0$ is unstable; otherwise it is stable. Substituting $\mathbf{s} = \bar{\mathbf{s}} + \delta\mathbf{s}$ into Eqs. (1) and (2) and employing the usual arguments, we conclude that the growth rate η satisfies the following equation

$$\begin{aligned} &\rho^2 c\eta^3 + \rho\left(\beta\dot{\gamma}^0 P_0 + k\xi^2 + cR_0\xi^2\right)\eta^2 + \left(-\beta\tau^0 P_0 + \rho c\left(f^0 S_0 + Q_0\right)\right. \\ &\left. + kR_0\xi^2\right)\xi^2\eta + k\left(f^0 S_0 + Q_0\right)\xi^4 = 0. \end{aligned} \tag{13}$$

Here

$$P_0 = -\frac{\partial\tau}{\partial\theta}|_{\mathbf{s}=\bar{\mathbf{s}}}, \quad Q_0 = \frac{\partial\tau}{\partial\gamma}|_{\mathbf{s}=\bar{\mathbf{s}}}, \quad R_0 = \frac{\partial\tau}{\partial\dot{\gamma}}|_{\mathbf{s}=\bar{\mathbf{s}}}, \quad S_0 = \frac{\partial\tau}{\partial\phi}|_{\mathbf{s}=\bar{\mathbf{s}}}, \tag{14}$$

and a superscript 0 on a variable signifies its value for the homogeneous solution at time t_0. For materials exhibiting strain hardening, strain-rate hardening and thermal softening, $P_0 \geqslant 0$, $Q_0 \geqslant 0$, $R_0 \geqslant 0$. We assume that $f^0 \geqslant 0$ and $S_0 \geqslant 0$. That is, the value of the internal variable increases with an increase in the plastic strain, and the shear stress required to plastically deform a material point increases with an increase in the value of the internal variable. Thus, the coefficients of η^3, η^2 and η° in Eq. (13) are non-negative, and a necessary condition for Eq. (13) to have a positive root is that the coefficient of η must be negative. Hence, if $P_0 = 0$, the homogeneous solution will be always stable.

In terms of nondimensional variables

$$\bar{\eta} = \frac{k\eta}{cQ_0}, \quad \bar{\xi} = \frac{k\xi}{c\sqrt{\rho Q_0}}, \quad I = \frac{cR_0}{k}, \quad J = \frac{\beta\tau^0 P_0 - \rho c f^0 S_0}{\rho c Q_0},$$
$$D = \frac{\beta k P_0 \dot{\gamma}^0}{\rho c^2 Q_0}, \quad E = 1 + \frac{f^0 S_0}{Q_0}, \tag{15}$$

Eq. (13) becomes

$$\bar{\eta}^3 + \left[D + (1+I)\bar{\xi}^2\right]\bar{\eta}^2 + \left(I\bar{\xi}^2 + 1 - J\right)\bar{\xi}^2\bar{\eta} + E\bar{\xi}^4 = 0. \tag{16}$$

For given values of t_0 and $\bar{\xi}$, Eq. (16) has three roots; the root with the largest positive real part will make the homogeneous solution most unstable; it is, henceforth, referred to as the dominant instability mode and denoted by $\bar{\eta}_d$. As is clear from previous similar studies, for a fixed t_0 (e.g. see Batra and Chen, 1999; Molinari, 1997; Chen and Batra, 1999), $\bar{\eta}_\mathrm{d}$ depends upon $\bar{\xi}$. We seek the wave number $\bar{\xi}_\mathrm{m}$ for which $\bar{\eta}_\mathrm{d}$ assumes the maximum value $\bar{\eta}_\mathrm{m}$. Thus $\bar{\eta}_\mathrm{m}$ and $\bar{\xi}_\mathrm{m}$ satisfy (16) and

$$\left(\frac{\mathrm{d}\bar{\eta}}{\mathrm{d}\bar{\xi}^2}\right)\Big|_{(\bar{\eta}=\bar{\eta}_\mathrm{m},\ \bar{\xi}=\bar{\xi}_\mathrm{m})} = 0. \tag{17}$$

Eqs. (16) and (17) give

$$\bar{\xi}_\mathrm{m}^2 = \bar{\eta}_\mathrm{m}\frac{(J-1) - (1+I)\bar{\eta}_\mathrm{m}}{2(I\bar{\eta}_\mathrm{m} + E)}, \tag{18}$$

and since $\bar{\xi}_\mathrm{m}^2 > 0$, therefore

$$0 \leqslant \bar{\eta}_\mathrm{m} \leqslant \frac{(J-1)}{(1+I)}. \tag{19}$$

Substitution for $\bar{\xi}_\mathrm{m}^2$ from (18) into the spectral Eq. (16) yields

$$4(I\bar{\eta}_\mathrm{m} + E)(\bar{\eta}_\mathrm{m} + D) = [(J-1) - (1+I)\bar{\eta}_\mathrm{m}]^2.$$

Thus, whenever

$$J > 1 + 2\sqrt{ED},$$

or

$$\frac{\beta\tau^0 P_0}{\rho c Q_0} - \frac{f^0 S_0}{Q_0} > 1 + 2\left[\left(1 + \frac{f^0 S_0}{Q_0}\right)\frac{\beta k P_0 \dot{\gamma}^0}{\rho c^2 Q_0}\right]^{1/2}, \tag{20}$$

Eq. (16) will have a solution $\bar{\eta}_m$ with a positive real part. Eq. (20) is the instability condition and generalizes Bai's (1982) work to materials with an internal variable. If $f^0 = 0$ or $S_0 = 0$, Eq. (20) reduces to Bai's (1982) instability criterion.

The smallest value of the average strain $\gamma_c = \dot{\gamma}^0\ t_0$ and hence of t_0 for which Eq. (16) has a root $\bar{\eta}_d$ with a positive real part is, henceforth, called the critical strain. The value of the critical strain can be computed from Eq. (20). In the results presented below, β is set equal to 1.0. For the four viscoplastic relations stated in Section 3 and the HY-100 steel for which material parameters are given in Section 4, Fig. 2 exhibits the relationship between the critical strain and the nominal strain-rate. It is clear that the critical strain is a monotonically increasing function of the nominal strain-rate for the Bodner–Partom relation, but is a monotonically decreasing function of the nominal strain-rate for the other three relations. For each one of these four relations, the dependence of the critical strain upon the average strain-rate is rather weak. For example, for the Bodner–Partom relation, the critical strain increases from 0.265 at $\dot{\gamma}^0 = 10^2$/s to 0.284 at $\dot{\gamma}^0 = 10^6$/s. In each case the critical strain coincided with the value of the average strain at which the shear stress peaked during homogeneous deformations of the body. That is, the strain evaluated from the instability condition

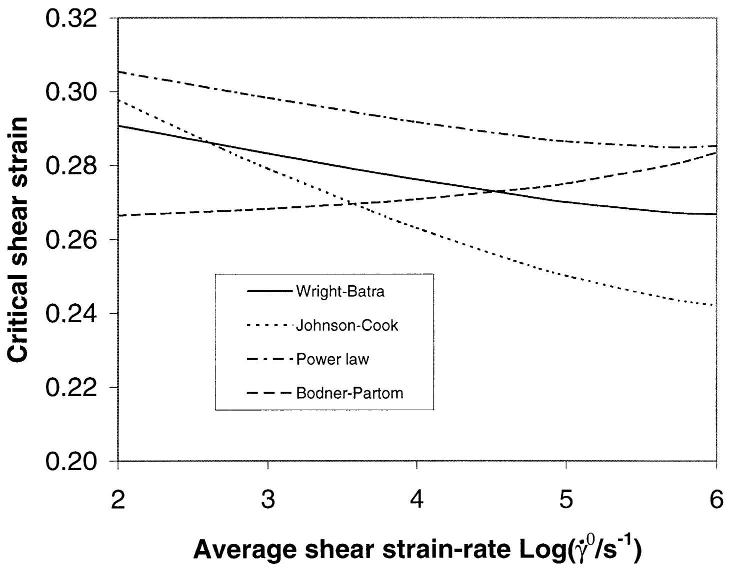


Fig. 2. Variation of the critical shear strain with the nominal strain-rate for the four viscoplastic relations.

$$\frac{\mathrm{d}\tau}{\mathrm{d}\gamma} \equiv \frac{\partial\tau}{\partial\gamma} + \frac{\partial\tau}{\partial\dot{\gamma}}\frac{\mathrm{d}\dot{\gamma}}{\mathrm{d}\gamma} + \frac{\partial\tau}{\partial\theta}\frac{\mathrm{d}\theta}{\mathrm{d}\gamma} = 0, \quad \frac{\mathrm{d}\theta}{\mathrm{d}\gamma} = \frac{\beta\tau}{\rho c}, \tag{21}$$

and the critical strain evaluated from the perturbation analysis were exactly the same. Note that for the homogeneous solution at a uniform strain-rate, the balance of linear momentum $(1)_1$ is identically satisfied, and the balance of internal energy $(1)_2$ reduces to Eq. $(21)_2$.

6. Shear band spacing

The dominant instability mode, $\bar{\eta}_{\mathrm{d}}$, depends upon the time t_0 (or the average strain $\gamma^0 = \dot{\gamma}^0 t_0$) when the perturbation is introduced and the wave number ξ. For a fixed t_0, one can compute the dominant instability mode as a function of the wave number ξ and find the supremum, η_{m}, of the real parts of the roots of Eq. (13). Henceforth, we denote the wave number corresponding to η_{m} by ξ_{m}; both η_{m} and ξ_{m} depend upon t_0. Fig. 3(a)–(d) exhibits, for $\dot{\gamma}_0 = 10^3$/s,the dependence of η_{m} and the corresponding wave length $L_{\mathrm{s}} = 2\pi/\xi_{\mathrm{m}}$ on the average strain γ^0 or equivalently time t_0 for the four viscoplastic relations. The average strains corresponding to the maximum value of η_{m} and the minimum value of L_{s} equal, respectively, (5.0, ∞), (0.85,0.75), (0.6, 0.5) and (2.8, ∞) for the Wright–Batra, power law, Bodner–Partom and the Johnson–Cook relations. Batra and Chen (1999) modeled the thermoviscoplastic response of titanium, 4340 steel and a S-7 tool steel by the power law, the Wright–Batra relation and the Johnson–Cook relation respectively. They found that for the two steels the minimum value of L_{s} occurred for very large values of the average strain at the instant the homogeneous solution was perturbed. However, for the titanium modeled by the power law, the minimum value of L_{s} corresponded to a moderate value of the average strain. Batra and Chen's results for the titanium agreed with those of Molinari (1997). Fig. 4 evinces, for the four viscoplastic relations, the dependence upon the nominal strain rate of the average strain corresponding to the maximum growth rate of the infinitesimal perturbation. Whereas for the Wright–Batra relation the average strain corresponding to the maximum growth rate of the perturbation drops rapidly with an increase in the nominal strain-rate, it is essentially unchanged for the other three viscoplastic relations.

Grady and Kipp (1987) have derived an expression for the approximate shear band spacing by accounting for the momentum diffusion due to unloading within bands. Wright and Ockendon (1996) postulated that the minimum spacing, L, between adjacent shear bands is given by

$$L = 2\pi/\xi_s\left(t_0^\eta\right) \tag{22}$$

where t_0^η corresponds to the time when $\eta_{\mathrm{s}}(t_0)$ is maximum. We note that Bai (1982) termed $1/\xi_{\mathrm{s}}(t_0^\eta)$ as the characteristic length. Molinari (1997) hypothesized that

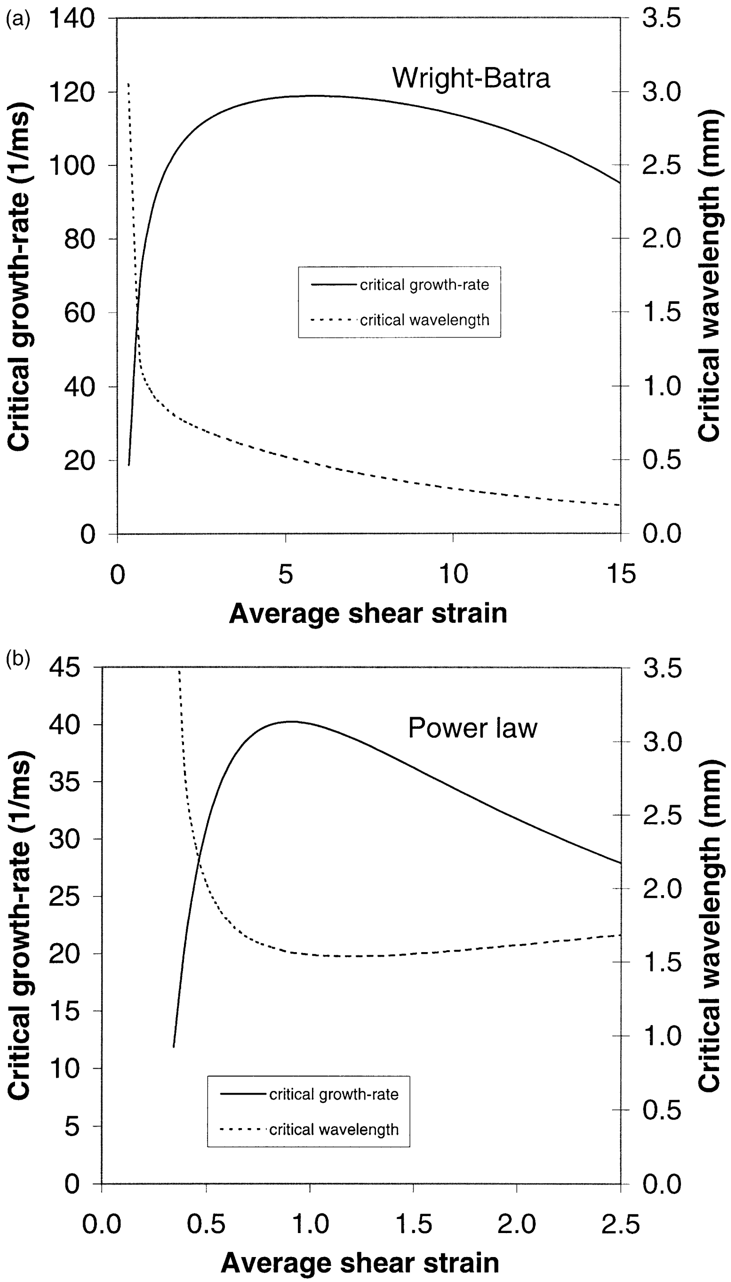
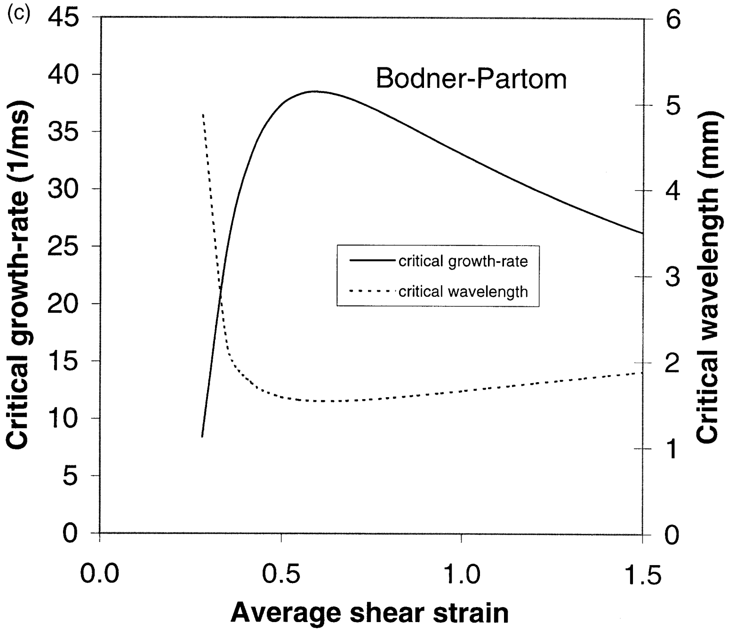


Fig. 3. The dependence, on the average shear strain when the homogeneous solution is perturbed, of the critical growth rate and the critical wavelength for the (a) Wright–Batra, (b) power law, (c) Bodner–Partom, and (d) Johnson–Cook relations (*continued on next page*).



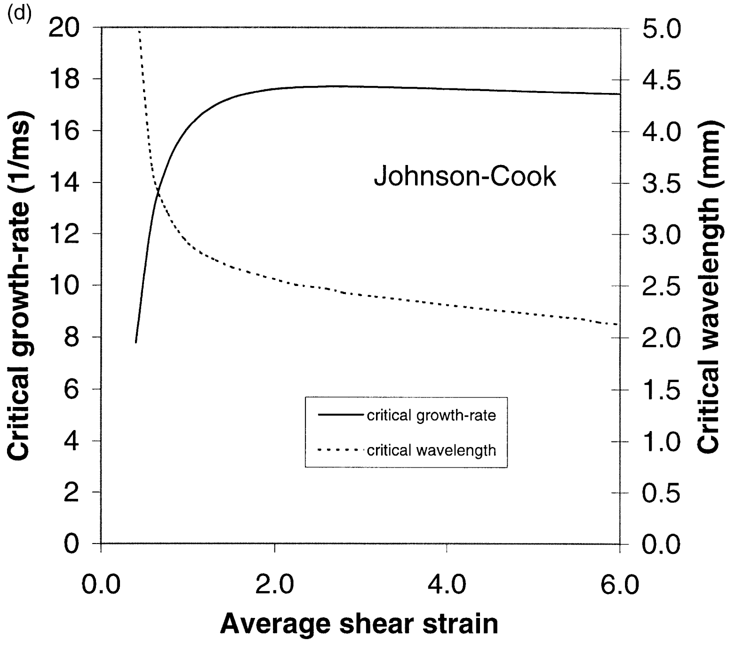


Fig. 3. (*Continued*)

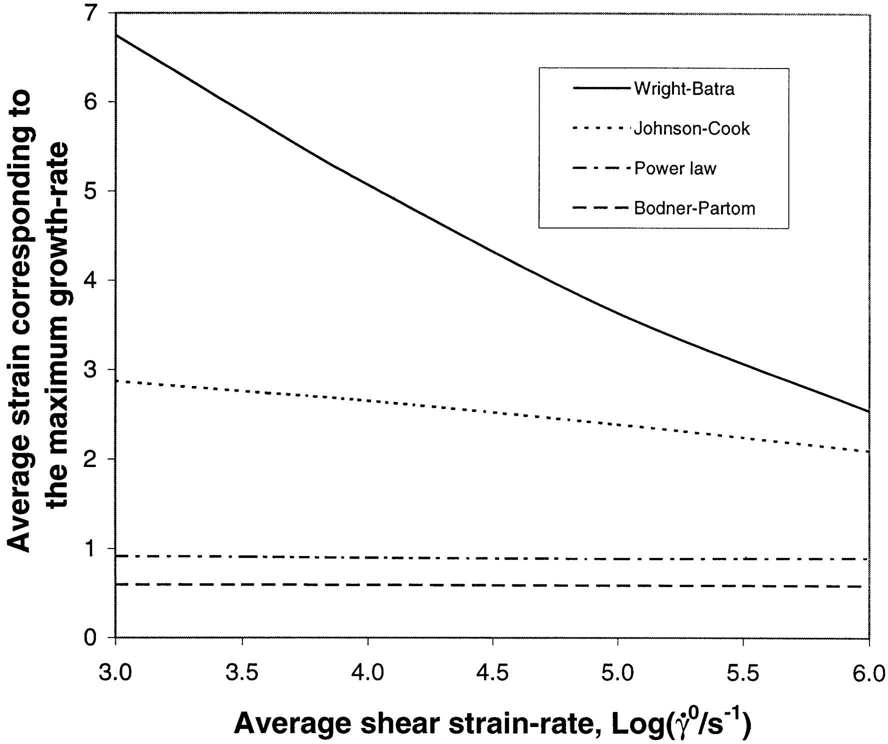


Fig. 4. Dependence upon the average strain rate of the average strain corresponding to the maximum growth rate of the infinitesimal perturbation superimposed upon a homogeneous solution.

$$\tilde{L} = \overset{\inf}{t_0 \geqslant 0}\,\frac{2\pi}{\xi_s(t_0)} \tag{23}$$

gives the minimum spacing between adjacent shear bands. For the titanium modeled by the power-law, Molinari showed through numerical computations that the two definitions (22) and (23) give essentially the same value of the shear band spacing. Batra and Chen's (1999) work indicates that these two definitions give quite different values of the shear band spacing when thermal softening of the material is modeled by an affine function of the temperature rise. Results plotted in Fig. 3(a)–(d) reveal that, according to the definition (23), the shear band spacing will be essentially zero for the Wright–Batra and the Johnson–Cook relations and will have a finite positive value for the other two viscoplastic relations. However, the definition (22) will yield a positive value of the shear band spacing for each one of the four viscoplastic relations and is adopted here. We note that Kwon and Batra (1988) and Batra and Kim (1990c) introduced a temperature perturbation with multiple cusps and numerically solved the initial-boundary-value problem. Kwon and Batra found that in a typical steel modeled by the Wright–Batra relation, a shear band formed at each trough in the cosine wave when the overall strain-rate was 500/s but at each crest for an average strain-rate of 50,000/s. They did not attempt to find the minimum spacing between adjacent shear bands.

Fig. 5 depicts the dependence of the shear band spacing upon the average strain-rate for the four viscoplastic relations. For each one of these relations, the shear band spacing rapidly decreases with an increase in the nominal strain-rate till $\dot{\gamma}^0 = 10^5$/s and subsequently decreases slowly. Whereas the four viscoplastic relations give widely different values of the shear band spacing at $\dot{\gamma}^0 = 10^3$/s, these differences virtually disappear for $\dot{\gamma}^0 \geqslant 10^5$/s. At nominal strain-rates of 3300/s, 10^4/s and 10^6/s, the computed shear band spacings in mm equal (0.473, 0.226, 0.011), (1.569, 0.687, 0.025), (2.022, 0.743, 0.047) and (2.45, 1.042, 0.033) respectively for the Wright–Batra, power law, Bodner–Partom and Johnson–Cook relations. Note that these values of the shear band spacing are for an infinite block and without the consideration of boundary conditions. For $\dot{\gamma}^0 = 10^6$/s, shear band spacings computed from the expressions given by Grady and Kipp (1987) and Wright and Ockendon

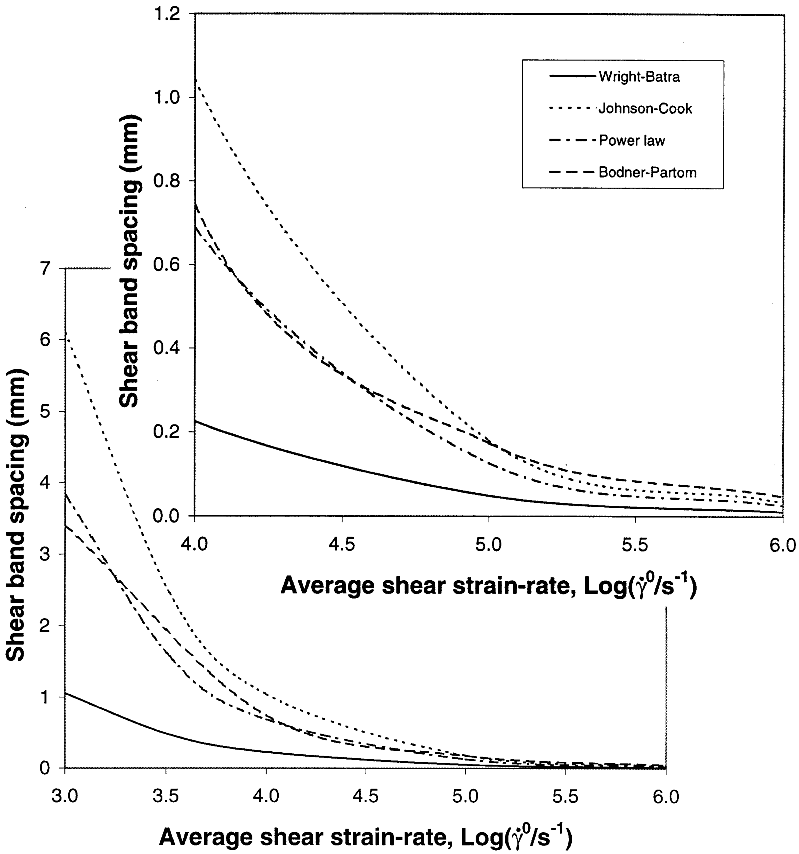


Fig. 5. The variation of the shear band spacing with the average strain-rate for the four viscoplastic relations.

(1996) equal 0.22 and 0.014 mm respectively. In an explosively loaded stainless steel cylinder, Nesterenko et al. (1995) estimated the strain-rate within a shear band to equal 10^4/s, and measured shear band spacing of about 1 mm.

7. Shear band width and energy dissipation

The initial-boundary-value problem formulated in Section 2 with the thickness variation $w(y)$ given by

$$w(y) = w_0\left[1 - 0.05\sin\frac{\pi}{2}\left(1 - \frac{y}{h}\right)\right], \quad 0 \leqslant y \leqslant h,$$

was analyzed by the finite element method for each one of the four viscoplastic relations. We used the code developed by Batra and Kim (1990a,b,c) and set the thickness at the center of the plate equal to 95% of that at the outer edge; the thickness varied sinusoidally between these two points. The region (0, $h = 2.5$ mm) was divided into 100 nonuniform elements with y-coordinate of nth node given by

$$y_n = h\left(\frac{n-1}{100}\right)^3, \quad n = 1, 2, ..., 101.$$

Following Batra and Kim (1992) we assume that a shear band initiates when the shear stress has dropped to 90% of its maximum value. Fig. 6 (a and b) shows the dependence, upon the nominal strain-rate, of the average shear strain, γ_{I}, corresponding to the maximum shear stress, and the average shear strain,$\gamma_{0.9}$, when a shear band initiates. As for the critical strain obtained with the perturbation analysis and plotted in Fig. 2, for the Bodner–Partom relation, both γ_{I} and $\gamma_{0.9}$ monotonically increase with an increase in the nominal strain-rate. For the other three viscoplastic relations, the critical strain gradually decreased initially with an increase in the nominal strain-rate and the dependence of both γ_{I} and$\gamma_{0.9}$ upon $\dot{\gamma}^0$ is not monotonic. It is interesting to note that for each one of the four viscoplastic relations the value of γ_{I} is lower than that of the critical strain at the same nominal strain-rate discussed in Section 5. This is because the critical shear strain is a material property but γ_{I} also depends upon the shape, size and the type of defect. Batra and Kim (1992) have delineated, through numerical experiments, the dependence of γ_{I} upon the defect size for twelve materials. They studied simple shearing deformations of these materials and modeled each one of them by the Johnson–Cook relation. Batra (1987) analyzed the effect of different amplitudes and shapes of initial perturbations upon the time of initiation of a shear band.

Marchand and Duffy (1988) conducted torsional tests on thin-walled steel tubes and stated that their measured band-width equaled the width of the intensely deformed region of uniform plastic strain. Batra and Kim (1992) equated the band-width to the width of the region over which the plastic strain was within 95% of its maximum value. Here we adopt the following definition: the bandwidth equals the

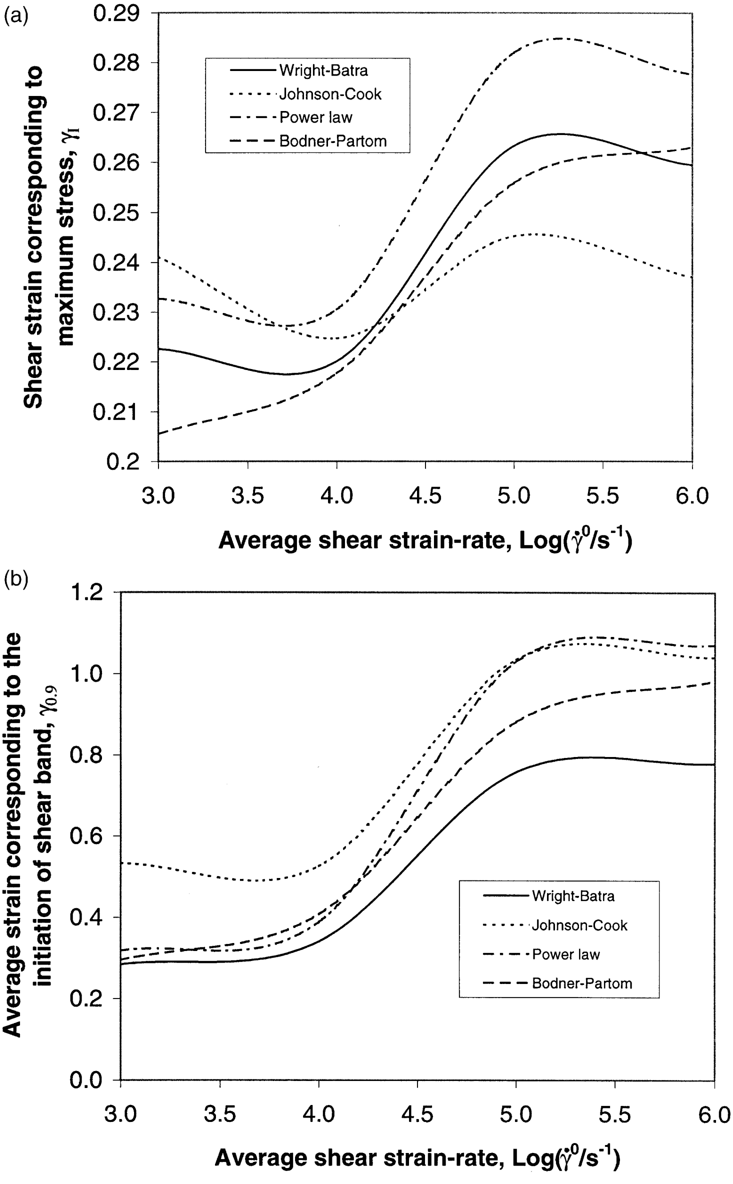


Fig. 6. For the four viscoplastic relations, dependence upon the average shear strain-rate of the shear strain corresponding to the maximum shear stress and that when the shear stress has dropped to 90% of its maximum value.

width of the region over which the plastic strain varies by no more than 10% of its maximum value. In Fig. 7, we have plotted the so computed bandwidth, δ, as a function of $\tau/\tau_{\max}$ for each one of the constitutive relations and a nominal strain-rate of 10^3/s. Whereas for the Wright-Batra and the Johnson-Cook relations, the

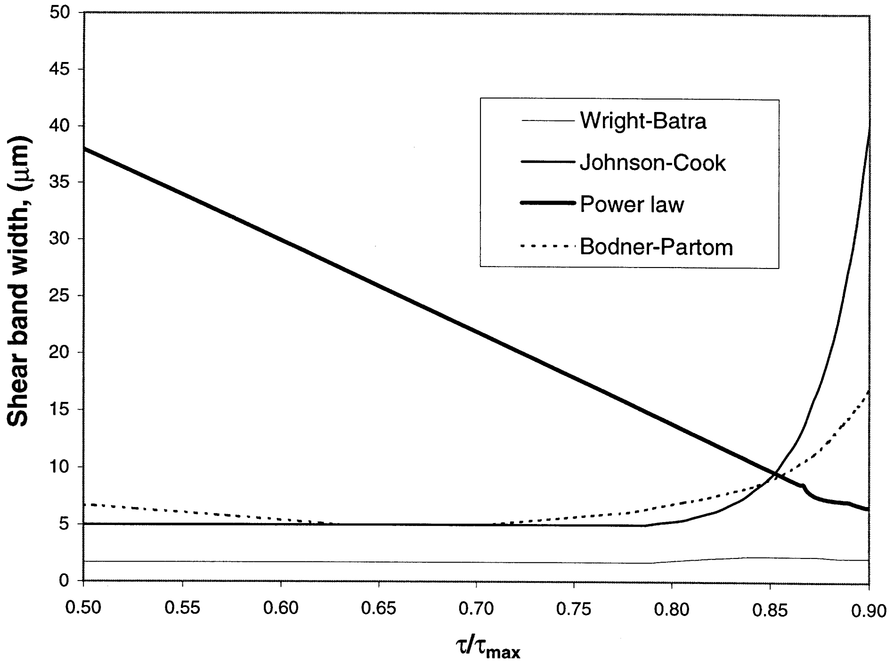


Fig. 7. For the four constitutive relations, the evolution of the shear band width during the time the shear stress is dropping.

band width very slowly decreases with the drop of the shear stress at the band center, for the power law it increases. For the Bodner–Partom relation, the band width increases slightly when the shear stress drops from 0.70 τ_{max} to 0.50 τ_{max}. Note that the horizontal axis in Fig. 7 may be regarded as a distorted time scale. Dilellio and Olmstead (1997) used asymptotic methods to analyze boundary layers near the edges of a specimen deformed in simple shear and modeled the material response by a power law type strain hardening and an exponential thermal softening relation. Their computed time-history of the evolution of the band width agrees qualitatively with the presently computed one for the power law.

For each one of the four viscoplastic relations, Fig. 8(a)–(d) exhibit the dependence of the computed band width upon the nominal strain-rate when $\tau/\tau_{max} = 0.9$, 0.8, 0.7 and 0.6. In each case, the band width first decreases with an increase in the nominal strain-rate from 10^2/s to about $10^{2.8}$/s and then increases. At $\tau/\tau_{max} = 0.9$, and $\dot{\gamma}^0 = 10^2$/s, the band width computed with the Wright–Batra relation is the minimum, δ_{min}, and that with the Johnson–Cook relation the maximum, δ_{max}, and $\delta_{max}/\delta_{min} \simeq 7$. For $\tau/\tau_{max} = 0.66$ and $\dot{\gamma}^0 = 1600$/s, Marchand and Duffy (1988) found that the band width varied between 20 μm and 55 μm around the circumference of the tube; our computations give $\delta = 2.3$, 5.5, 14 and 6 μm for the Wright–Batra, Johnson–Cook, power law and the Bodner–Partom relation.

The nondimensional rate of energy, $\dot{E}$, dissipated per unit volume of the shear banded material is given by

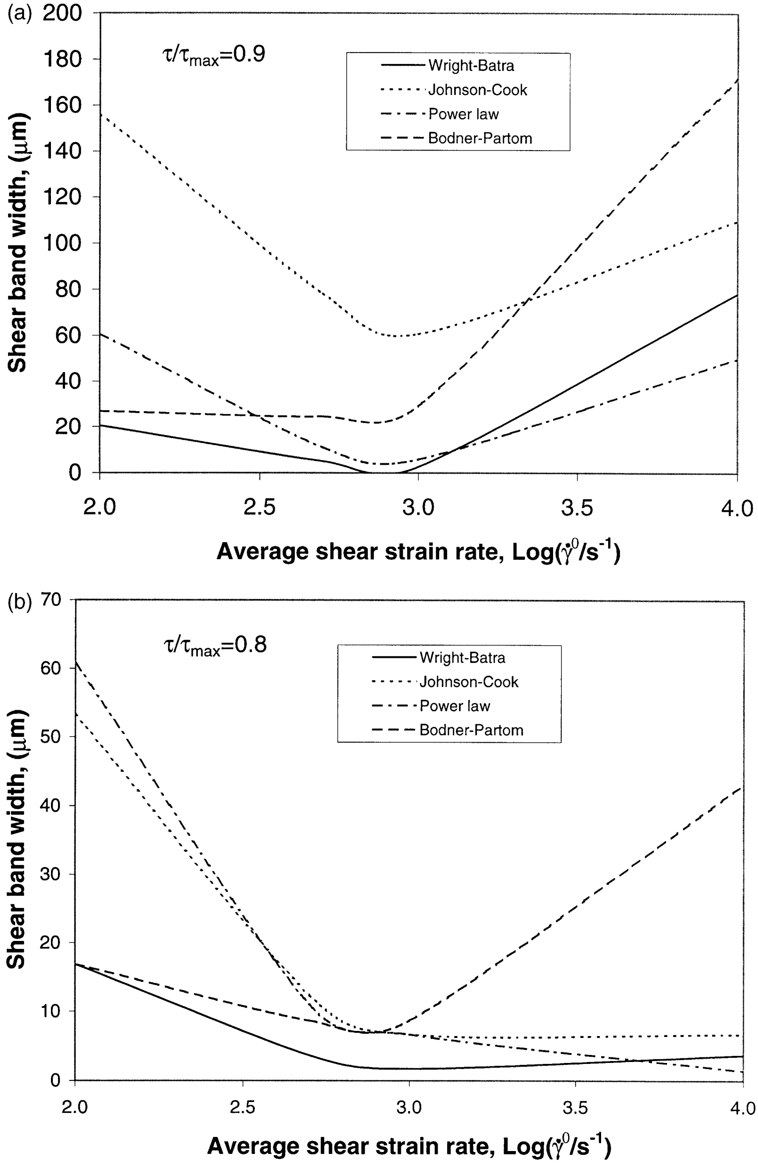


Fig. 8. For the four viscoplastic relations, the dependence of the shear band width upon the nominal strain-rate at four instants of time (*continued on next page*).

$$\dot{E} = \frac{2}{\delta \tau_0 \gamma_{\text{avg}}} \int_0^{\delta/2} \tau \dot{\gamma} \mathrm{d}y.$$

For $\dot{\gamma}^0 = 10^3$/s, Fig. 9 shows, for the four viscoplastic relations, the energy dissipation rate $\dot{E}$ vs. $\tau/\tau_{\max}$. The maximum value of $\dot{E}$ occurs at $\tau/\tau_{\max} = 0.87$, 0.62,

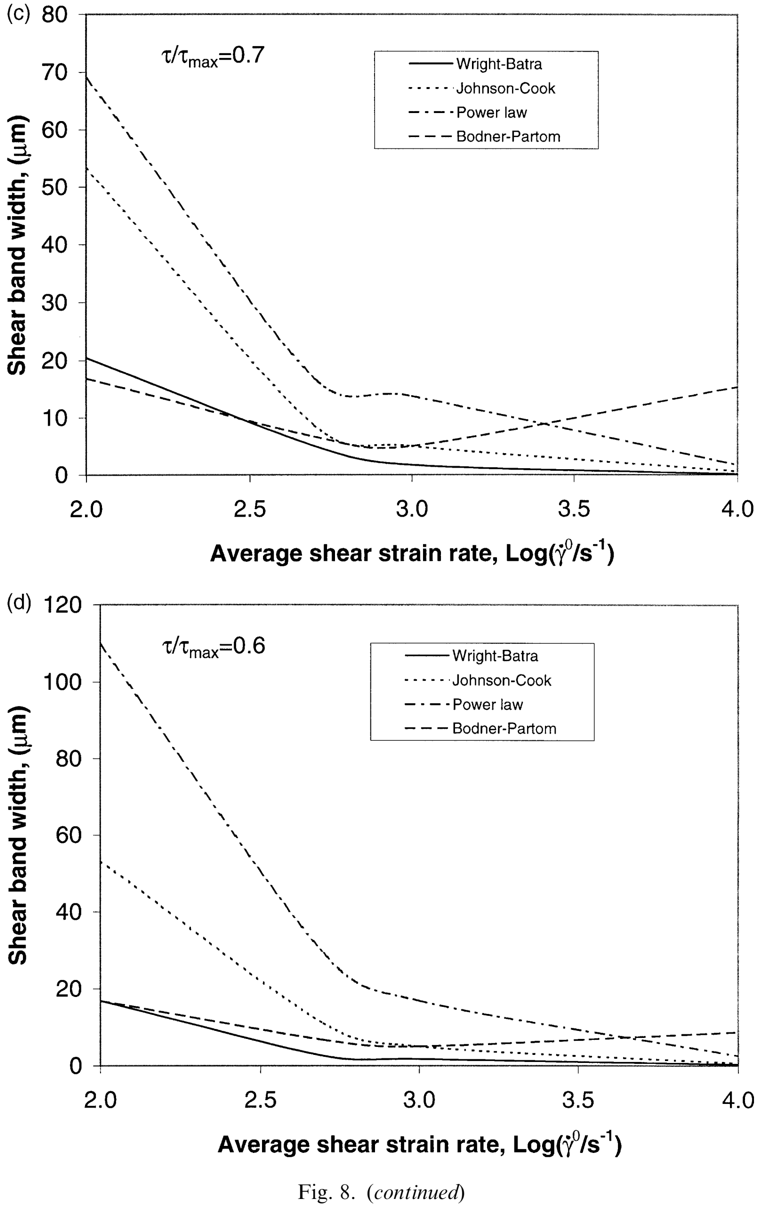


Fig. 8. (*continued*)

0.91 and 0.72 for the Wright–Batra, Johnson–Cook, power law and the Bodner–Partom relations. For the power law and the Wright–Batra relation, the strain-rate oscillated for $\tau/\tau_{\max} \leqslant 0.85$ and 0.82 respectively, and values of $\dot{E}$ could not be satisfactorily computed.

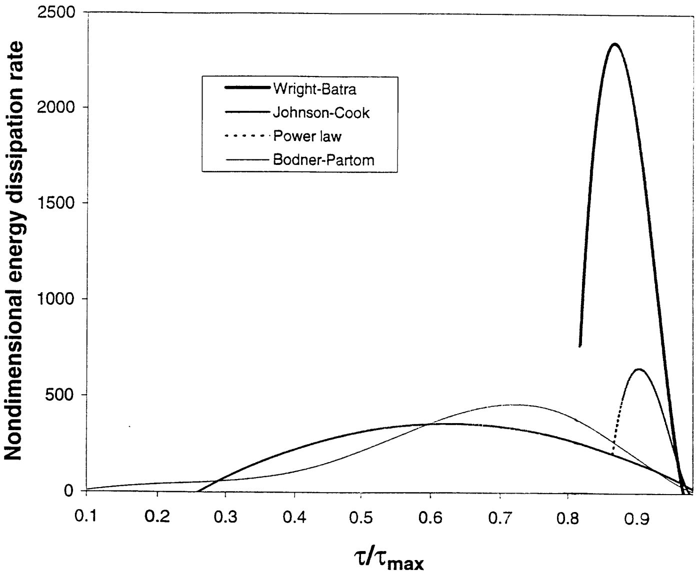


Fig. 9. For the four viscoplastic relations, the evolution of the normalized rate of energy dissipated per unit volume of the shear banded material during the time the shear stress is dropping.

8. Effect of specimen height

An advantage of numerical experiments over physical experiments is that finite homogeneous deformations can be produced in large specimens. Higher values of the specimen height should reduce the influence of boundary conditions exerted by the grips of the loading device. In the numerical experiments described below, the nominal strain-rate is kept fixed at 10^3/s. For the Bodner–Partom relation, Fig. 10(a) and (b) exhibit for $h = 2.5$, 5 and 25 mm the development of the shear band width and the normalized density of the energy dissipation rate during the post localization process; similar results were obtained with the Johnson–Cook relation. In order to significantly reduce the effect of the finite element mesh on the computed results, 200 elements were used for $h = 25$ mm. Thus the size of the smallest element in the mesh is essentially the same for all three values of h. For each value of h, the band width first decreases with a drop in the shear stress at the band center, and eventually begins to gradually increase due to the heat conducted outwards from the band center. The energy dissipation rate, $\dot{E}$, per unit volume of the shear banded material first increases with a drop in the shear stress at the band center, attains its maximum value, and then decreases. A noteworthy observation is that $\dot{E}$ is maximum and the band width is minimum at $\tau/\tau_{\max} \simeq 0.78$, 0.70 and 0.48 respectively for $h = 2.5$, 5.0 and 25 mm; the corresponding values of the band width are 5.79, 5.12 and 8.32 μm. It suggests that the shear band has fully developed when $\dot{E}$ reaches its maximum value which supports Grady and Kipp's (1987) hypothesis.

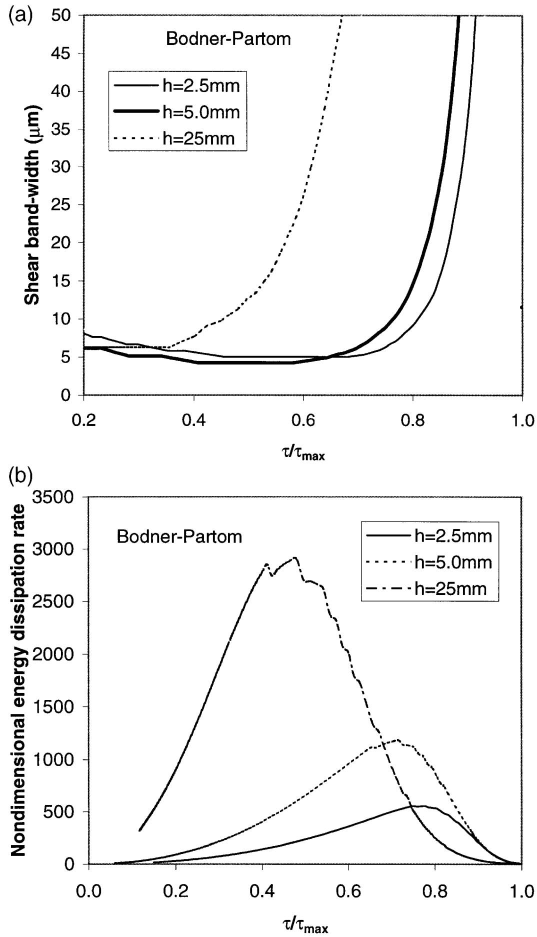


Fig. 10. For the Bodner–Partom relation, the evolution of (a) the band width and (b) the normalized density of the energy dissipation rate for $h = 2.5$, 5.0 and 25.0 mm during the time the shear stress is dropping.

Recalling that the nominal shear strain-rate $\dot{\gamma}_{avg}$ is kept fixed, an increase in the value of h requires that the shearing speed, v_0, at the outer edge of the specimen be also increased. Once the shear band has fully developed, the particle speed increases from 0 at $y = 0$ to v_0 at the edge of the shear band. Even though the fully developed band is wider for the higher value of h, the proportional increase in the band width is much less than the increase in the height of the specimen. Thus, as shown in Fig. 11a the strain-rate at the band center when $\tau/\tau_{max} = 0.6$ increases sharply with an increase in the value of h. But the shear stress at the band center for $h = 2.5$ mm and 25 mm is about the same, e.g. see Fig. 11(b). Thus, $\dot{E} \simeq \tau\dot{\gamma}$ increases sharply with an increase in the value of h. When $\tau/\tau_{max} = 0.6$, the total energy dissipated per unit

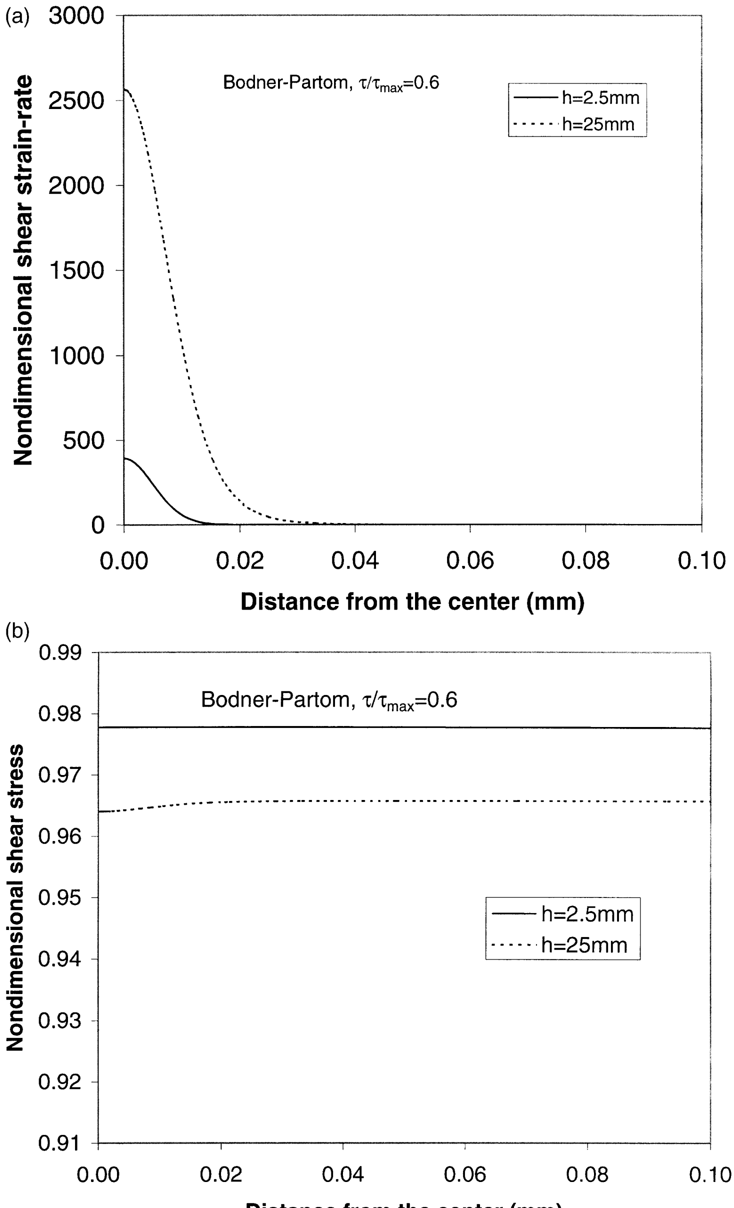


Fig. 11. For the Bodner–Partom relation and for $h = 2.5$, 5 and 25 mm, spatial variation of (a) the strain-rate, and (b) the shear stress when $\tau/\tau_{\max} = 0.6$, and (c) the time-history of the energy dissipation rate (*continued on next page*).

volume of the shear banded material equals 5.14×10^7, 3.77×10^8, and 1.4×10^8 J/m^3 respectively for $h = 2.5$, 5.0 and 25 mm. Even though the maximum value of $\dot{E}$ is very high for $h = 25$ mm as compared to that for $h = 2.5$ mm, the total energy dissipated is

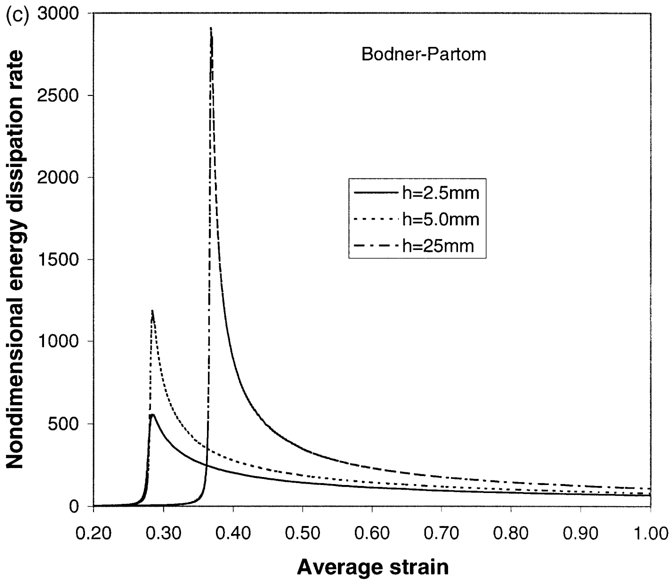


Fig. 11. (*continued*)

less because the time taken for the development of the shear band drops significantly with an increase in the value of h, see Fig. 11(c). For a shear band width of 7 μm [cf. Fig. 10(a)], and $h=2.5$, 5 and 25 mm the energy dissipated per unit surface area of the shear band equals respectively 1.8, 1.32, 0.49 kJ/m^2 which is an order of magnitude lower than the 44.7 kJ/m^2 predicted by Grady and Kipp (1987) for the 4140 steel. They assumed the material to thermally soften but ignored the strain and strain-rate hardening of the material.

The integration of Eq. $(1)_2$ over the shear banded region $(0, \delta/2)$ and the use of the divergence theorem gives

$$\int_0^{\delta/2} \rho w c \dot{\theta} \mathrm{d}y = k\theta_{,y}|_{y=\delta/2} + \int_0^{\delta/2} w\tau\dot{\gamma}\mathrm{d}y. \tag{24}$$

We denote the value of the term on the left-hand side of (24) by I, and those of the first and the second terms on right-hand side of (24) by II and III respectively. The evolution of these terms during the post-localization process is plotted in Fig. 12; note that II < 0 and its magnitude is plotted in Fig. 12. It is clear that as the deformation localizes, the rate of energy conducted out of the shear band edge increases and eventually becomes a significant part of the rate of energy dissipated within the shear band. At this stage the rate of temperature rise becomes vanishingly small. For $\tau/\tau_{\max}=0.6$, the rate of heat conducted out of the edge of the shear band equals about 85% of the rate of the energy dissipated within the band.

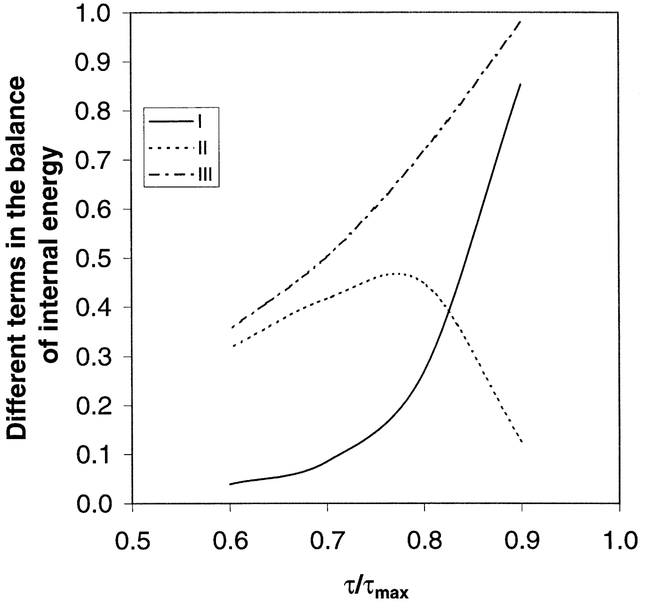


Fig. 12. For the Bodner–Partom relation, the evolution during the postlocalization process of different terms in the balance of internal energy Eq. (24).

9. Conclusions

We have studied thermomechanical deformations of a steel block deformed in simple shear. The thermoviscoplastic response of the material has been modeled by four relations due to Wright and Batra, Bodner and Partom, and Johnson and Cook and also by power law. The values of material parameters in these viscoplastic relations were determined by solving initial-boundary-value problems corresponding to homogeneous simple shearing deformations of the block at a nominal strain-rate of 3300/s. These values were such that, in each case, the computed shear stress vs. shear strain curve closely matched the experimental curve obtained by Marchand and Duffy in torsional tests on a thin-walled tube made of a HY-100 steel. The values of material parameters can not be uniquely determined since the underlying nonlinear coupled thermomechanical problem does not have a unique solution.

The stability of the homogeneous solution has been studied by using the perturbation technique. For each one of the four viscoplastic relations, the critical shear strain, i.e. the average shear strain at which the homogeneous solution first becomes unstable equaled that at which the shear stress attained its peak value. With an increase in the average strain-rate, the critical shear strain monotonically increased for the Bodner–Partom relation but decreased for the other three relations. In each case the change in the critical strain was rather small when the nominal strain-rate was increased from 10^2 to 10^6/s.

For the same HY-100 steel, the four viscoplastic relations gave quite different values of the shear band spacing and the band width. In each case the shear band spacing decreased monotonically with an increase in the average strain-rate. At the instant when the shear stress had dropped to 60% of its maximum value, for each viscoplastic relation, the band width decreased with an increase in the average strain rate from 10^2 to about $10^{2.8}$/s and then increased for the Bodner–Partom relation but decreased for the other three relations when the strain-rate was further increased. For strain rates between 10^2 and $10^{2.8}$/s, the band width is less sensitive to the strain-rate for the Wright–Batra and the Bodner–Partom relations and quite sensitive to the nominal strain-rate for the other two relations. For an average strain rate of 3300/s, the normalized rate of energy dissipated per unit volume of the shear banded material attains its maximum value of 2300, 300, 650, and 450 at $\tau/\tau_{\max}$ = 0.87, 0.62, 0.91 and 0.72 respectively, for the Wright–Batra, the Johnson–Cook, the power law and the Bodner–Partom relations.

The maximum strain-rate at the band center and the average strain-rate within the band increased sharply when the height of the specimen was changed from 2.5 to 25 mm, and the corresponding minimum band width increased from about 5 to 6.25 μm. The numerical experiments reveal that the energy disipation rate per unit volume of the shear banded material is maximum at the instant the band has fully developed as indicated by a little change in the band width during subsequent deformations.

For the Bodner–Partom relation, at $\tau/\tau_{\max} = 0.6$, the rate of heat conducted out of the edge of the shear band equals nearly 85% of the rate of the energy dissipated within the band.

Acknowledgements

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