



# Elements of Continuum Mechanics

**Romesh C. Batra**

Virginia Polytechnic Institute and State University

Batra is the Clifton C. Garvin Professor, Department of Engineering Science and Mechanics, at Virginia Polytechnic Institute and State University. He earned his B.Sc. at Panjabi (now called Thapar) University, Patiala, India, his M.A.Sc. at the University of Waterloo, Canada, and his Ph.D. at the Johns Hopkins University in the U.S.

August 2005 • 308 pages • Hardback • ISBN: 1563476991  
List Price: \$79.95 • **AIAA Member Price: \$54.95**

## ABOUT THE BOOK

As the ever-growing capabilities of numerical tools are being used to simulate how fluids and solids respond to high temperatures and severe loads, a fundamental understanding of the theory of large coupled thermo-mechanical deformations will play an ever-increasing role. This requires a sound understanding of the basic principles of non-linear continuum mechanics such as those given in this text book. This book differs from several others in the market in that it provides numerous example problems from every day life which should help develop a better understanding of a nonlinear theory. The book is oriented more towards engineers than mathematicians.

After briefly reviewing the mathematical preliminaries in Chapter 2, the theory of large deformations of a continuum is developed in subsequent chapters. Special emphasis is placed on constitutive relations. Theories developed in earlier chapters are illustrated through solutions of several nonlinear and linear problems in latter chapters.

## CONTENTS

Preface

Acknowledgments

### Chapter 1 Introduction

1.1 What is Mechanics?

1.2 Continuum Mechanics

1.3 An Example of an Ad-Hoc Approach

### Chapter 2 Mathematical Preliminaries

2.1 Summation Convention, Dummy Indices

2.2 Free Indices

2.3 Kronecker Delta

2.4 Index Notation

2.5 Permutation Symbol

2.6 Manipulations with the Indicical Notations

2.7 Translation and Rotation of Coordinate

Axes

2.8 Tensors

2.9 The Divergence Theorem

2.10 Differentiation of Tensor Fields

References

Exercises

### Chapter 3 Kinematics

3.1 Description of Motion of a Continuum

3.2 Referential and Spatial Descriptions

3.3 Displacement Vector

3.4 Restrictions on Continuous Deformation of a Deformable Body

3.5 Material Derivative

3.6 Finding Acceleration of a Particle from a Given Velocity Field

3.7 Deformation Gradient

3.8 Strain Tensors

3.9 Principal Strains

3.10 Deformation of Areas and Volumes

3.11 Mass Density, Equation of Continuity

3.12 Rate of Deformation, Strain-Rate Tensor, Spin

3.13 Polar Decomposition

3.14 Infinitesimal Deformations

3.15 Infinitesimal Deformations

Superimposed upon Finite

Deformations

3.16 Volumetric and Deviatoric Strains

3.17 Transformation of Tensors Under a Change of Bases

3.18 Plane Strain Deformation

Appendix A: Solution of a Cubic Equation

References

Exercises

### Chapter 4 The Balance Laws, Stress Tensors

4.1 Kinetics of a Continuous Media

4.2 Traction Boundary Conditions

4.3 The Nominal Stress Tensor

4.4 Transformation of Stress Tensors Under the Rotation of Axes

4.5 Principal Stresses; Maximum Shear Stress

4.6 Relations Among Stress Tensors for Infinitesimal Deformations

4.7 Plane Stress

4.8 Deviatoric Stress, von-Mises Stress

4.9 Balance of Energy

Order 24 hours a day at [www.aiaa.org](http://www.aiaa.org)

Publications Customer Service

P.O. Box 960

Herndon, VA 20172-0960

Phone: 800/682-2422; 703/661-1595

Fax: 703/661-1501

E-Mail: [warehouse@aiaa.org](mailto:warehouse@aiaa.org) • Web: [www.aiaa.org](http://www.aiaa.org)



## CONTENTS CONTINUED

4.10 Entropy Inequality, The Clausius-Duhem Inequality  
4.11 Summary of Equations Governing Deformations of a Body  
4.12 Nonuniqueness of Solutions for Static Problems  
Appendix B: The Transport Theorem  
Exercises  
**Chapter 5 Constitutive Relations**  
5.1 Introductory Remarks  
5.2 Thermoelastic Material  
5.3 Principle of Material Objectivity  
5.4 Linear Constitutive Relations for Finite Deformations of a Thermoelastic Body  
5.5 Isotropic Thermoelastic Materials  
5.6 Comparison of Results from Four Linear Constitutive Relations in Isotropic Finite Elasticity  
5.7 Transversely Isotropic Thermoelastic Materials  
5.8 Orthotropic Thermoelastic Materials  
5.9 Coincidence of Principal Axes of Stress and Strain in Isotropic Elastic Materials  
5.10 Coincidence of Principal Axes of Stress and Strain in Transversely Isotropic Elastic Materials  
5.11 Incompressible Elastic Materials  
5.12 Comparison of Results from Constitutive Relations  
5.13 Constitutive Relations for Infinitesimal Deformations of Elastic Materials  
5.14 Constitutive Relations for Special Isotropic Nonlinear Elastic Materials  
5.15 Infinitesimal Deformations Superimposed upon Finite Deformations of an Isotropic Elastic Body  
5.16 Constitutive Relations for Plane Deformations of a Thermoelastic Body  
5.17 Thermoviscoelastic Materials  
5.18 Summary  
References  
Exercises

**Chapter 6 Torsion of a Circular Cylinder**  
6.1 Torsion of a Linear Elastic Circular Cylinder  
6.2 Torsion of a Second-Order Elastic Circular Cylinder  
6.3 Infinitesimal Twist of a Finitely Stretched Circular Cylinder  
6.4 Finite Torsion of a Circular Cylinder  
Appendix C: A Uniqueness Theorem  
References  
Exercise  
**Chapter 7 Fluid Flow**  
7.1 Steady Flow Between Two Parallel Plates  
7.2 Steady Isothermal Flow of an Incompressible Fluid Down an Inclined Plane  
7.3 Steady Flow of an Incompressible Fluid in a Horizontal Circular Pipe  
Exercise  
**Chapter 8 Bending of Beams**  
8.1 Bending of a Rectangular Beam  
8.2 Bending of a Nonlinear Elastic Rectangular Beam  
8.3 Airy Stress Function for Bending of a Beam  
Exercises  
**Chapter 9 Wave Propagation**  
9.1 Singular Surface  
9.2 Kinematics of a Singular Surface  
9.3 Acceleration Waves in Linear Elasticity  
9.4 Progressive Waves  
9.5 Incompressible Linear Elastic Materials  
9.6 Acceleration Waves in Nonlinear Elastic Bodies  
9.7 Infinitesimal Deformations Superimposed upon Finite Deformations  
Exercises  
**Chapter 10 Spherical and Cylindrical Pressure Vessels**  
10.1 Radial Expansion of a Spherical Pressure Vessel  
10.2 Radial Expansion of an Incompressible Hookean Sphere with Shear Modulus a Function of Radius  
10.3 Radial Expansion of a Cylindrical Pressure Vessel  
10.4 Radial Expansion of an Inhomogeneous and Incompressible Hookean Cylinder  
10.5 Finite Radial Expansion of a NeoHookean Cylinder  
Index  
Supporting Materials

## ALSO FROM AIAA

**Introduction to Aircraft Flight Mechanics: Performance, Static Stability, Dynamic Stability, and Classical Feedback Control** • Thomas R. Yechout, with Steven L. Morris, David E. Bossert, and Wayne F. Hallgren • 2003, 649 pp, Hardback • ISBN: 1563475774 • **Analytical Mechanics of Space Systems** • Hanspeter Schaub and John L. Junkins • 2003, 600 pp, Mixed media • ISBN: 1563475634 • **Composite Materials for Aircraft Structures, Second Edition** • A. A. Baker, S. Dutton, D. Kelly • 2004, 400 pp, Hardback • ISBN: 1563475405

**ORDER 24 HOURS A DAY AT [WWW.AIAA.ORG](http://WWW.AIAA.ORG)!**